

Topic 1 - Basic Counting

Statistics for Managers

June 3, 1999

Multiplication of Choices:

Suppose a choice or possibility consists of two steps, the first of which can be made in m ways and the second in n ways. The total number of choices or possibilities can be made in $m \times n$ ways.

Example 1. Toss two coins once

Toss two coins once, how many ways this can occur?

First coin has two possibilities, $m = 2$

Second coin has two possibilities, $n = 2$

Total number possibilities: $2 \times 2 = 4$

Example 2. Toss a coin and a die once

Toss a coin and a die once, how many ways can this occur?

Coin has two possibilities, $m = 2$

Die has six possibilities, $n = 6$

Total number possibilities: $2 \times 6 = 12$

Example 3. Full page advertisements

Two companies place full page advertisements (first-come-first-served) in trade magazine to be in a) back of cover page, b) front of back page, c) back of back page. How many ways can this occur?

First company has three choices, $m = 3$

Second company has two choices, $n = 2$

Total number outcomes: $3 \times 2 = 6$

Generalized Multiplication of Choices

If a choice or possibility consists of k steps, the first of which can be made in n_1 ways and the second in n_2 ways, etc. Then the total number of choices or possibilities can be made in $n_1 \times n_2 \times \dots \times n_k$ ways.

Example 4. Full page advertisements

Three companies place full page advertisements (first-come-first-served) in trade magazine to be in a) back of cover page, b) front of back page, c) back of back page. How many ways can this occur?

First company has three choices, $n_1 = 3$

Second company has two choices, $n_2 = 2$

Third company has one choice, $n_3 = 1$

Total number outcomes: $3 \times 2 \times 1 = 6$

Factorial notation:

$$n! = n(n-1)(n-2)\dots(2)(1)$$

Example 5. Six people sit

How many ways can six people sit around a round table?

Answer is $6!$, but you can also use the multiplication of choices.

First person has six places to sit, $n_1 = 6$

Second person has five places to sit, $n_2 = 5$

Third person has four places to sit, $n_3 = 4$

Fourth person has three places to sit, $n_4 = 3$

Fifth person has two places to sit, $n_5 = 2$

Sixth person has one place to sit, $n_6 = 1$

Total number of possibilities: $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 = 6!$

Restricted First Choice

Example 6. One choice must be a woman

Three of ten employees, four female and 6 male, are to be chosen for an overseas assignment. At least one choice must be a woman. How many ways can this occur?

First choice must be from the group of women $n_1 = 4$

Second choice is from the remaining men and women $n_2 = 9$

Third choice is from the remaining men and women $n_3 = 8$

Total number ways: $4 \times 9 \times 8 = 288$

Number of Permutations

n object can be ordered in $n!$ ways.

Example 7. Five marine signal flags

Five marine signal flags can be ordered to transmit $5! = 120$ different messages.

Number of Ordered Choices

r objects can be selected from a total of n object in

$$n \times (n-1) \times \cdots \times (n-r+1)$$

ordered ways.

Example 8. Two from five marine signal flags

Two marine signal flags can be selected from 5 flags to transmit $5(5 - 2 + 1) = 5 \times 4 = 20$ different messages.

One may get the same answer using multiplication of choices.

First choice from five flags, $n_1 = 5$

Second choice from remaining flags, $n_2 = 4$

Total number different messages: $5 \times 4 = 20$

Homework 1 Six out of 46 LOTTO GAME

The NJ official website states that of number of ways the lottery can occur is 9,366,819. Verify the number.

Source: <http://www.state.nj.us/lottery/pk6rule.htm>

Number of Combinations

r objects can be selected from a total of n objects in $n(n-1)\cdots(n-r+1)/r!$ unordered ways.

" n choose r ":

$$\begin{aligned}\binom{n}{r} &= {}_n C_r = \frac{n!}{(n-r)!r!} \\ &= \frac{n(n-1)\cdots(n-r+1)}{r!}\end{aligned}$$

Example 9. Computers shipped

The number of combinations of two computers shipped from a stock of nine computers is 36.

$$\begin{aligned}\binom{9}{2} &= {}_9 C_2 = \frac{9!}{(9-2)!2!} \\ &= \frac{9(9-1)\cdots(9-2+1)}{2!} \\ &= 36\end{aligned}$$

Homework 2 Coin Tossing

Flip four coins. Use the number of combinations formula to find how many ways two heads can appear.

Compounded Choices Involving Number of Combinations

Example 10. Defective computers

One of 10 computers is defective, the number of ways the defective computer can be shipped when three are shipped is 36:

Use multiplication of choices.

First choice is to select the defective computer, $n_1 = 1$

Second choice is to select two computers from the remaining group, $n_2 = {}_9C_2 = 36$

Total number of ways: $1 \times 36 = 36$

Example 11. Two women

Two teams, three to a team, made up from ten employees, 4 female and 6 male, are to be chosen for an overseas assignment. One team must have exactly two women. How many ways can this occur?

First team of two women and one man, $n_1 = {}_4C_2 \times 6 = 6 \times 6 = 36$

Second team of three from the remaining people, $n_2 = {}_7C_3 = 35$

Total number of ways: $36 \times 35 = 1260$

Homework 3 Two women and two men

Two teams, three to a team, made up from ten employees, 4 female and 6 male, are to be chosen for an overseas assignment. One team must have exactly two women and one team must have exactly two men. How many ways can this occur?