
Project 1 – Truth Sampling

Statistics for Managers

Name: _____

Student Number: _____

Date: _____

Part A

Suppose Y and T are random variables taking values 0 or 1 only. Suppose further that

$$P(Y = 1) = p; P(Y = 0) = 1 - p$$

$$P(T = 0) = \frac{1}{2}; P(T = 1) = \frac{1}{2}$$

$$P(Y = 1 | T = 1) = p_1; P(Y = 0 | T = 1) = 1 - p_1$$

$$P(Y = 1 | T = 0) = \frac{1}{2}; P(Y = 0 | T = 0) = \frac{1}{2}$$

Prove that

$$p = \frac{1}{2} p_1 + \frac{1}{4}$$

Therefore,

$$p_1 = 2p - \frac{1}{2}$$

Hint: Use the following relationships

$$P(Y = 1) = P(\{Y = 1\} \cap \{T = 1\}) + P(\{Y = 1\} \cap \{T = 0\})$$

$$P(\{Y = 1\} \cap \{T = 1\}) = P(Y = 1 | T = 1)P(T = 1)$$

$$P(\{Y = 1\} \cap \{T = 0\}) = P(Y = 1 | T = 0)P(T = 0)$$

Answer

Part B

Compose a question of a sensitive nature to ask a person. For example, you could ask them if that had ever cheated on a school exam; alternatively, if they had ever stolen property; alternately, if they had ever cheated on their spouse – or whatever interests you. Take a random sample of 35 people and ask them to give only one answer to your question. To conceal their individual circumstance you will allow them to tell the truth or give a random answer.

All you will record is their answer to your question. However, they will toss a coin twice to decide if they will tell the truth or give a random answer. You will not observe their coin tossing activity at all. Moreover, they will answer your question only once.

Instruct them to go away, take their own coin, and toss it once, if it is heads, then they must tell the truth. However, if it is tails, then they must toss the coin again – if heads this time they answer “yes”, if tails they answer “no” – all regardless of the truth.

Using the results in Part A, let

$$Y = \begin{cases} 1, & \text{if subject answers "yes"} \\ 0, & \text{if subject answers "no"} \end{cases}$$

Let

$$T = \begin{cases} 1, & \text{if first toss is heads i.e. "tell the truth"} \\ 0, & \text{if first toss is tails i.e. "random reply"} \end{cases}$$

Record all observations on Y , let p denote the proportion of “1’s”. Clearly, you want the proportion of “1’s” when the first toss is heads (which you don’t see), meaning they are telling the truth. Denote this unknown proportion by p_1 . Calculate and report the estimate of p_1 using the results of Part A.

Answer:

Your Question:

Data:

Subject ID	Answer (1=yes 0=no)	Subject ID	Answer (1=yes 0=no)	Subject ID	Answer (1=yes 0=no)	Subject ID	Answer (1=yes 0=no)	Subject ID	Answer (1=yes 0=no)
1		8		15		22		29	
2		9		16		23		30	
3		10		17		24		31	
4		11		18		25		32	
5		12		19		26		33	
6		13		20		27		34	
7		14		21		28		35	

Estimate:

$p =$ _____

$p_1 =$ _____

