

## Topic 10b - The Exponential Distribution

### Statistics for Managers

June 3, 1999

The exponential random variable appears in many important applications. In business, the time between customers arriving for service is an exponential random variable. In fact, the time between Poisson events is an exponential random variable. Thus, with every Poisson random variable, there is an associated exponential random variable. Vice versa, if the time between events is an exponential random variable, then the number of events occurring is a Poisson random variable. The exponential random variable has a continuous distribution that has a simple formula.

### Exponential Distribution

Let  $Y$  be an exponential random variable. The probability density function is

$$f(y) = \beta e^{-\beta y}, y > 0$$

The mean and variance are

$$E(Y) = \frac{1}{\beta}$$

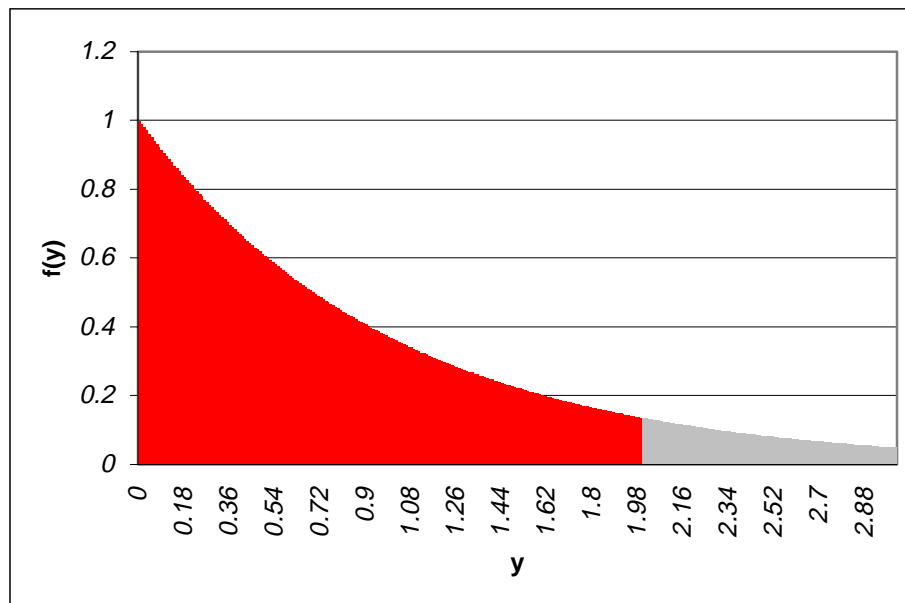
$$Var(Y) = \frac{1}{\beta^2}$$

$$\beta > 0$$

The cumulative distribution function is

$$F(y) = P(Y \leq y) = 1 - e^{-\beta y}$$

## Histogram of the Exponential Distribution



### Example 1.

The time between machine failures at an industrial plant has an exponential distribution with an average of 2 days between failures.

Suppose a failure has just occurred at the plant. Find the probability that the next failure won't happen in the next 5 days.

Answer: Let  $Y$  denote the time between accidents. The mean time to failure (MTF) is 2 days. Thus  $\beta = 1/2$ . We want  $P(Y > 5) = 1 - P(Y \leq 5) = 1 - F(5)$ .

$$F(5) = 1 - e^{-(0.5)(5)} = 1 - e^{-2.5}$$

$$1 - F(5) = e^{-2.5} = 0.082085$$

If we would expect most of the values of the distribution to fall within  $\mu \pm 3\sigma$ , give an upper bound for the longest time between failure that we expect to see.

Answer:  $\mu = E(Y) = 2.0$ ;  $\sigma^2 = \text{Var}(Y) = 2.0^2$ ;  $\sigma = 2.0$ .

Thus, the upper bound is  $\mu + 3\sigma = 2.0 + 3(2.0) = 8.0$  days.

## Homework 1.

### *05:40 PM ET 07/27/99 1,000 Asteroids May Threaten Earth*

WASHINGTON (AP) \_ Up to 1,000 asteroids are large enough and close enough to pose a potential risk to Earth, but the chances of a collision are extremely remote, astronomers say. Researchers attending a conference on asteroids and comets at Cornell University said Tuesday that new data suggests there are 500 to 1,000 nearby asteroids big enough to cause planet-wide damage if they collided with the Earth. David Rabinowitz of the Jet Propulsion Laboratory said that there are probably 2,000 to 4,000 asteroids of about one-third of a mile in diameter that cross the orbital path of the Earth. These would be big enough to cause extensive regional damage if they hit the Earth, but probably would not have the planet-wide effects of the larger space rocks. Asteroid impacts have scarred the Earth and moon for billions of years, but the impact of very large asteroids \_ the so-called planet killers \_ are extremely rare. The last major extinction, which killed off the dinosaurs, occurred 65 million years ago when an asteroid five to eight miles in diameter smashed the Earth. Experts estimate that a mile-wide asteroid could possibly hit the Earth only about once every 100,000 to 1 million years. But for right now, there are no known big asteroids heading for the Earth. Paul Chodas of JPL said that through a worldwide effort, astronomers have now identified about 15 percent to 20 percent of the predicted number of asteroids dangerous to the Earth. He said the goal is to find at least 90 percent of the expected number. Once an asteroid is found, astronomers can monitor its orbit, or path around the sun, and determine if it poses any risk to Earth. Chodas said there currently are no looming threats from outer space.

**Assume the mile-wide asteroid arrival time is an exponential random variable with mean 100,000 years. Stating your assumptions, find the probability that such an asteroid will hit the earth in the next millennium.**