

## Topic 11 – Breakeven Analysis

### Statistics for Managers

June 3, 1999

## Preliminary Analysis Involving the Normal Distribution

### Example 1. Find z values that correspond to a given probability

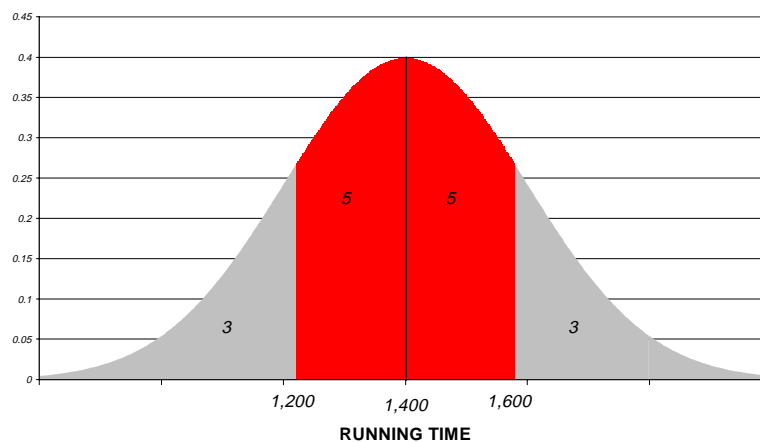
Find the value  $z$  such that  $P(Z \leq z) = 0.813$ .

From the standard normal probability table, the answer is approximately 0.89.

### Example 2. Using betting odds to estimate standard deviation

Suppose the running time of a type of machine is known to be a normal random variable. It is desired to know the mean and standard deviation, however, no hard data is available. Instead, the supervisor is asked to guess what the mean and standard deviation are from his long-time experience. The supervisor guesses the mean is about 1400 hours, but has no idea what a standard deviation is.

You divide the interval around 1400 into 1,200 and 1,600 hours. Using betting odds, you elicit from the supervisor what the odds are that the number of hours will lie between 1,200 and 1,600 hours. The supervisor says the odds are 5 to 3 (same as 10 to 6).



The value of  $z$ , such that the standard normal random variable is less than  $13/16 \cong 0.813$ , is approximately 0.89, as seen in the prior example. However, the Z score, using the unknown standard deviation, of 1,600 hours is 0.89. That is,

$$z = \frac{1,600 - 1,400}{\sigma} = 0.89$$

$$0.89\sigma = 200 \text{ hr}$$

$$\sigma = 225 \text{ hr}$$

## The Normal Probability Density Function and Cost-Volume-Profit Analysis

Quantitative methods are useful for analyzing in advance the likely consequences of alternative product, price, or output strategies management is currently considering. Cost-Volume-Profit Analysis, a.k.a. breakeven analysis, allows management to determine in advance the effects that certain contemplated decisions or expected state of nature will have on revenues, costs, and therefore profits. Of the two profits variables, revenue and costs, management has considerable less control over revenue. Thus, the estimation of revenues is a good example of decision making under conditions of risk, where management can specify a distribution of revenues.

Suppose a company with these financial data is considering a new product:

Selling price	\$7.50
Variable cost/unit	\$4.50
Fixed cost/year	\$1,500,000

In cost-volume-profit analysis, the breakeven point is the point at which total cost equals total revenue, and can be express as:

$$\text{Breakeven point (in units)} = \frac{\text{Total fixed costs}}{\text{Price/unit} - \text{Variable cost/unit}}$$

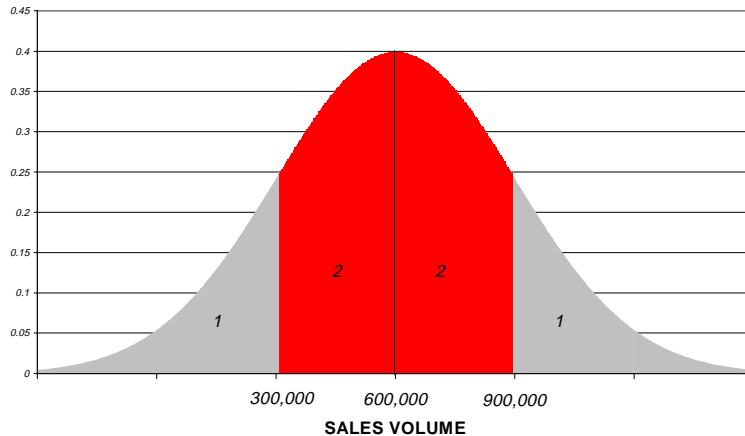
**Total fixed cost = indirect costs or costs that do not vary with volume**

**Price/unit = selling price for the item**

**Variable cost/unit = direct costs per unit and which are constant per unit over a large range of volume**

$$\begin{aligned} \text{Breakeven point (in units)} &= \frac{\$1,500,000}{\$7.50 - \$4.50} \\ &= \frac{\$1,500,000}{\$3.0} \\ &= 500,000 \text{ units annually} \end{aligned}$$

Now suppose the sales manger estimates that the mean sales would be 600,000 units for the coming year. When asked about the variability of sales volume, she indicates the chances are 2 to 1 (same as 4 to 2) that sales will be within 300,000 of the mean she has estimated.



From the above figure, we can see that the area to the left of 900,000 units is  $5/6$ , or 0.8333. In the standard normal distribution table, the value  $z$  for which  $P(Z \leq z) = 0.8333$  is about 0.97. Thus the  $Z$  score, using the unknown standard deviation, of 900,000 units is 0.97:

$$z = \frac{900,000 - 600,000}{\sigma} = 0.97$$

$$0.97\sigma = 300,000 \text{ units}$$

$$\sigma = 309,278 \text{ units}$$

**Example 3. What is the probability of at least breaking even?**

The breakeven-point is 500,000 units with a  $Z$  score of

$$z = \frac{500,000 - 600,000}{309,278} = -0.323$$

If  $Y$  represents sales volume, then the probability of breaking even is

$$P(Y > 500,000) = P(Z > -0.323) = P(Z \leq 0.323) = 0.626$$

**Example 4. What is the probability that profits would be at least \$500,000?**

Since the new product profit margin is \$3, for profits to be at least \$500,000, sales will have to be at least  $\$500,000/\$3 = 166,667$  units above the breakeven point of 500,000 units. The  $Z$  score of 666,667 is

$$z = \frac{666,667 - 600,000}{309,278} = 0.216$$

Using the standard normal table, we see that the probability that profits would be at least \$500,000 is

$$P(Y > 666,667) = P(Z > 0.216) = 1 - P(Z \leq 0.216) = 1 - .586 = 0.414$$

**Example 5. What are the chances of loosing \$250,000 or more?**

To lose that much money sales would have to be at least  $\$250,000/\$3 = 83,333$  units below the 500,000 breakeven point, that is no higher than 416,667 units. The Z score of 416,667 is -0.593. Using the standard normal table, we see that the probability of loosing \$250,000 or more is

$$P(Y \leq 416,667) = P(Z \leq -0.593) = 1 - P(Z \leq 0.593) \cong 1 - 0.722 = 0.278$$

## Homework 1 Break-Even Analysis

In marketing, W. Shih<sup>1</sup> developed two practical decision criteria using break-even analysis.

**Decision Rule A:** Market the new product if the chance is better than 50% that demand  $D$  will exceed the break-even point  $BE$ , i.e., if  $P(D \geq BE) > 0.5$ .

**Decision Rule B:** For a specific level of risk  $p$ , where  $0 \leq p \leq 1$ , market the new product if  $P(D \geq BE) > p$ .

Note that the decision rule A is a special case of decision rule B, with specified level of risk  $p = 0.5$ . Suppose a company utilizes break-even analysis to decide whether to market a new type of ceiling fan. From past experience, the company knows that the number of ceiling fans of this type sold per year follows a normal distribution with  $\mu = 4,000$  and  $\sigma = 500$ . Marketing researchers have also determined that the company needs to sell 3,500 units to break even for the year.

- a. According to decision rule A, should the company market the new ceiling fan?
- b. Use your knowledge of the normal distribution to show that if  $P(D \geq BE) > 0.5$ , then it must be true that  $\mu \geq BE$ .
- c. Suppose that the maximum level of risk the company is willing to tolerate is  $p = 0.8$ . Use decision rule B to arrive at a decision.
- d. Refer to part c. Find the value of  $BE$  such that the probability of at least breaking even is equal to the specific level of risk,  $p = 0.8$ .

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<sup>1</sup> W. Shih, "A general decision model for cost-volume-profit analysis under uncertainty: A reply." *The Accounting Review*, Vol. 56, No. 2, 1981, pp. 404-408.