Topic 15 - Sample Mean and Sample Variance

Statistics for Managers

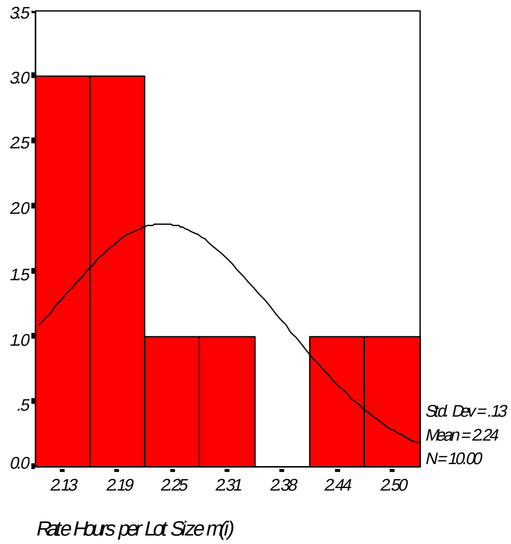
June 3, 1999

Example Dataset: Westwood Company

Westwood manufactures standard wall clocks for non-industrial consumption. Wholesalers order the clocks in Lot Sizes. Westwood is repeating a study about how many Man-Hours it takes to manufacture recent Lot Sizes for costing purposes and for comparing the efficiency of its manufacturing process to historical records. Several years ago, when Westwood originally did this study, it was determined that it should take on average 2 man-hours to produce one clock. Management is concerned the production hours are now longer since new workers do not have good job skills.

Obs (i)	*Lot Size s(i)*	*Man-Hours h(i)*	*Ratio y(i)*
1	*30*	*75*	*2.50*
2	*20*	*50*	*2.50*
3	*60*	*128*	*2.13*
4	*80*	*170*	*2.13*
5	*40*	*87*	*2.18*
6	*50*	*108*	*2.16*
7	*60*	*135*	*2.25*
8	*30*	*69*	*2.30*
9	*70*	*148*	*2.11*
10	*60*	*132*	*2.20*

Sample Histogram



Sample Mean

The <u>mean</u> is the "center of gravity" of the mass in a histogram.

It measures the location of a set of observations.

It can be positive, zero, or negative.

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

Sample Variance

The <u>variance</u> is the "moment of inertia" of the mass in the histogram.

It measures the central tendency of a set of observations.

If the variance is small, all the mass is tightly concentrated around the mean.

On the other hand, if the variance is large, all the mass is sparsely dispersed around the mean.

It can be positive or zero only.

$$s^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}$$

The square root of the variance is denoted as the <u>standard deviation</u>.

The values of the standard deviation and the mean can be compared since they are on the same numerical scale.

$$s = \sqrt{s^2}$$

Computational Formula for Sample Variance

This formula concerns the <u>numerator</u> of the variance.

$$SS_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$= \sum_{i=1}^{n} (y_i)^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

Example: Westwood

Lot Size		Man-Hours		Ratio	
Mean	50	Mean	110.2	Mean	2.245762
Standard Error	6.146363	Standard Error	12.25452	Standard Error	0.046092
Median	55	Median	118	Median	2.1875
Mode	60	Mode	#N/A	Mode	2.5
Standard Deviation	19.43651	Standard Deviation	38.7522	Standard Deviation	0.145755
Sample Variance	377.7778	Sample Variance	1501.733	Sample Variance	0.021244
Kurtosis	-1.06661	Kurtosis	-1.10941	Kurtosis	0.070688
Skewness	-0.11349	Skewness	-0.10764	Skewness	1.188185
Range	60	Range	120	Range	0.385714
Minimum	20	Minimum	50	Minimum	2.114286
Maximum	80	Maximum	170	Maximum	2.5
Sum	500	Sum	1102	Sum	22.45762
Count	10	Count	10	Count	10
Confidence Level(95.0%)	13.90405	Confidence Level(95.0%)	27.72168	Confidence Level(95.0%)	0.104267

EXCEL Calculations

Obs (i)	Lot Size s(i)	Man-Hours h(i)	Ratio y(i)	y-sq(i)
1	30	75	2.5000000	6.2500000
2	20	50	2.5000000	6.2500000
3	60	128	2.1333333	4.5511111
4	80	170	2.1250000	4.5156250
5	40	87	2.1750000	4.7306250
6	50	108	2.1600000	4.6656000
7	60	135	2.2500000	5.0625000
8	30	69	2.3000000	5.2900000
9	70	148	2.1142857	4.4702041
10	60	132	2.2000000	4.8400000
		SUM	22.4576190	50.6256652

Applying Formulas

$$n = 10$$

$$\sum_{i=1}^{n} y_i = 22.39095$$

$$\sum_{i=1}^{n} (y_i)^2 = 50.29678$$

$$\bar{y} = \frac{22.39095}{10} = 2.2391$$

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 = 50.29678 - \frac{(22.39095)^2}{10} = 0.16131581$$

$$s^2 = \frac{0.16131581}{9} = .017923979$$

$$s = \sqrt{.017923979} = .1339$$

Normal Q-Q Probability Plot

- **The Normal Q-Q Probability Plot of the man-hour rate of clocks manufactured at Westwood connotes <u>non-normality</u>!**